

Modern Physics Letters A
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GAMOW-TELLER SUM RULE IN RELATIVISTIC NUCLEAR MODELS

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Relativistic corrections are investigated to the Gamow-Teller(GT) sum rule with respect to the difference between the β_- and β_+ transition strengths in nuclei. Since the sum rule requires the complete set of the nuclear states, the relativistic corrections come from the anti-nucleon degrees of freedom. In the relativistic mean field approximation, the total GT strengths carried by the nucleon sector is quenched by about 12% in nuclear matter, while by about 8% in finite nuclei, compared to the sum rule value. The coupling between the particle-hole states with the nucleon-antinucleon states is also discussed with the relativistic random phase approximation, where the divergence of the response function is renormalized with use of the counter terms in the Lagrangian. It is shown that the approximation to neglect the divergence, like the no-sea approximation extensively used so far, is unphysical, from the sum-rule point of view.

1. Introduction

The sum rules play an important role in a wide range of physics. Since they are derived from the fundamental principles of the theory, their violation leads us to the reconstruction of the theory. Recently, precise analyses of experiment have been performed on the sum rule with respect to the difference between the β_- and β_+ Gamow-Teller(GT) transition strengths in ^{90}Zr . It has been shown that the GT sum rule value is quenched by $12\% \pm 6\%$ ¹, not including an overall normalization uncertainty in the GT unit cross section of 16%. The analyses have been performed in the excitation-energy region up to 50 MeV. Experimentally, it is not clear yet that the GT sum rule is violated, mainly because of the uncertainty in the GT unit cross section. It is important, however, for nuclear physics to study theoretically whether or not the GT sum rule is violated, since it should be hold, if the nucleus is a non-relativistic quantum mechanical system composed of nucleons. The study of the violation is also important for other several areas of physics. For example, information of nuclei is necessary for neutrino physics or supernova physics, where the dominant processes are governed by the GT and Fermi transitions. The pion condensation in neutron star can not be also discussed without the knowledge on the violation of the GT sum rule². Correct understanding of nuclei makes it possible

to discuss the GT strength in different situations.

Theoretically, it is pointed out that the GT sum rule is violated owing to the non-nucleonic degrees of freedom in nuclei, that is the Δ degrees of freedom. The Δ is an excited state of the nucleon, whose mass is 1323 MeV. The Δ -hole states are excited with additional GT strengths to the sum rule value. While the sum rule value is proportional to the difference between the neutron and the proton number, the total GT strength of the Δ -hole states is proportional to the number of nucleons. Therefore, if there is a repulsive coupling of the nucleon particle-hole states with the Δ -hole ones, their small mixing yields a considerable reduction of the GT strength in a low excitation-energy region³. Indeed, the coupling force is predicted to be repulsive. Recent calculations have shown that 19(14)% of the sum rule value is taken by the Δ -hole states, using the coupling constant of Chew-Low (the constituent quark) model⁴.

There are more naive corrections to the GT sum rule of the nucleonic degrees of freedom in the non-relativistic theory. In nuclei, relativistic corrections have been believed to be about 10% for a long time, since the nuclear Fermi velocity is estimated to be about 0.27 from the nuclear density in non-relativistic models. In the case of the sum rule, this correction is related to a requirement of the complete set of nuclear states. In a relativistic framework, the complete set is composed of the nucleon(N) and the anti-nucleon(\bar{N}) states. Since $N\bar{N}$ states are at the time-like region above 1 GeV, the GT strengths in the lower energy region should be reduced. The sum rule value may not be changed, but a part of the GT strengths are taken by $N\bar{N}$ states, which can not be excited through usual charge-exchange reactions. This means the violation of the GT sum rule in the nucleonic degrees of freedom⁵.

The purpose of the present paper is to review recent work on the relativistic corrections to the GT sum rule[5-11]. It will be shown that naive relativistic corrections reduce the GT strengths in the low excitation-energy region by about 12% in nuclear matter with the normal density and about 8% in finite nuclei. We will also examine effects of the coupling between the particle-hole states and the nucleon-antinucleon states in the random phase approximation by assuming relativistic Landau-Migdal parameters. The coupling requires the renormalization of the divergence in the relativistic response function. A model to renormalize the divergence is presented. It will be also shown that the so-called no-sea approximation¹², which is employed frequently in the literature^{5,8,9,13}, is unphysical, from the sum-rule point of view.

2. Gamow-Teller Sum Rule

Let us briefly mention the GT sum rules in the non-relativistic and relativistic theory. They are the model-independent sum rules with regard to the difference between the total β_- and β_+ transition strengths. The sum rule values are the same in both cases, and given by the difference between the neutron number N and the proton number Z of the system.

2.1. Gamow-Teller sum rule in the non-relativistic theory

In the non-relativistic theory, the GT sum rule is obtained by calculating¹⁴

$$S_{\text{nonrel}} = \sum_n \{ \langle 0 | Q_+ | n \rangle \langle n | Q_- | 0 \rangle - \langle 0 | Q_- | n \rangle \langle n | Q_+ | 0 \rangle \}, \quad (1)$$

where Q_{\pm} denotes the β_{\pm} transition operators, respectively,

$$Q_{\pm} = \sum_i^A \sigma_{yi} \tau_{\pm i} \quad \tau_{\pm} = (\tau_x \pm i\tau_y) / \sqrt{2}. \quad (2)$$

With use of the closure property of the intermediate states, the sum is expressed in terms of the commutator, so that the sum value is calculated to be

$$S_{\text{nonrel}} = \langle 0 | [Q_+, Q_-] | 0 \rangle = 2(N - Z). \quad (3)$$

Thus, the sum rule is just the result of the commutation relation between the isospin operators. Hence, any non-relativistic model should satisfy it.^a

If there is no ground state correlations in $N > Z$ closed subshell nuclei, as sometimes assumed for ^{90}Zr and ^{208}Pb , we have

$$Q_+ | 0 \rangle = 0. \quad (4)$$

Then the sum rule value is exhausted by the proton particle and neutron hole states excited through the β_- transitions,

$$\sum_n \langle 0 | Q_+ | n \rangle \langle n | Q_- | 0 \rangle = 2(N - Z). \quad (5)$$

2.2. Gamow-Teller sum rule in relativistic theory

In terms of the nuclear field $\psi(\mathbf{x})$, the excitation operators of the β_{\pm} transitions are written as

$$F_{\pm} = \int d^3x \bar{\psi}(\mathbf{x}) \Gamma_{\pm} \psi(\mathbf{x}), \quad (6)$$

where Γ_{\pm} stands for

$$\Gamma_{\pm} = \gamma_5 \gamma_y \tau_{\pm}. \quad (7)$$

The GT sum is described as

$$S_{\text{rel}} = \sum_n \{ \langle 0 | F_+ | n \rangle \langle n | F_- | 0 \rangle - \langle 0 | F_- | n \rangle \langle n | F_+ | 0 \rangle \}. \quad (8)$$

With use of the closure property of the intermediate states, the above equation is expressed by the commutator, as in the non-relativistic theory,

$$S_{\text{rel}} = \langle 0 | [F_+, F_-] | 0 \rangle. \quad (9)$$

^aIt should be noted that the sum rule value is given as $3(N - Z)$ in the literature¹⁴, where the excitation operator is defined by $\sum \sigma(\tau_x \pm i\tau_y)/2$, instead of Q_{\pm} .

The commutator is calculated as

$$\begin{aligned} [F_+, F_-] &= \int d^3x d^3x' (\gamma_0 \Gamma_+)_{\alpha\alpha'} (\gamma_0 \Gamma_-)_{\beta\beta'} \left[\psi_\alpha^\dagger(\mathbf{x}) \psi_{\alpha'}(\mathbf{x}), \psi_\beta^\dagger(\mathbf{x}') \psi_{\beta'}(\mathbf{x}') \right] \\ &= 2 \int d^3x \psi^\dagger(\mathbf{x}) \tau_z \psi(\mathbf{x}), \end{aligned} \quad (10)$$

by using the fact that

$$\{\psi_\alpha^\dagger(\mathbf{x}), \psi_{\alpha'}(\mathbf{x}')\} = \delta_{\alpha\alpha'} \delta(\mathbf{x} - \mathbf{x}'). \quad (11)$$

Finally we obtain the GT sum rule in the relativistic theory as

$$S_{\text{rel}} = 2(N - Z). \quad (12)$$

Thus, the sum rule value is the same as in the non-relativistic theory. It should be noted, however, that Eq.(11) holds in including the anti-nucleon degrees of freedom, and the closure property used in Eq.(9) requires $N\bar{N}$ states in addition to particle-hole states. If we neglect the $N\bar{N}$ states, the sum value of the nucleon sector must be reduced by the relativistic effect, compared to $2(N - Z)$.

3. Relativistic mean field approximation

So far, relativistic corrections have been estimated with use of the p/M expansion of the free field. Recent development of relativistic nuclear models for the past 30 years, however, make it possible to estimate the corrections consistently with the nuclear wave function without the expansion^{15,16}.

In the present section, we estimate relativistic corrections in the mean field approximation. First, we study the GT transition in nuclear matter, assuming that the nucleons and antinucleons are bounded in Lorentz scalar U_s and vector U_0 potentials. Next, we will discuss the relativistic corrections in finite nuclei, according to the more detailed mean field approximation.

In nuclear matter in Lorentz scalar and vector potentials, the mean field is given by the free field, but with the effective nucleon and anti-nucleon mass $M^* = M - U_s$,

$$\psi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^{3/2}} \sum_{\alpha} \left(u_{\alpha}(\mathbf{p}) \exp(i\mathbf{p} \cdot \mathbf{x}) a_{\alpha}(\mathbf{p}) + v_{\alpha}(\mathbf{p}) \exp(-i\mathbf{p} \cdot \mathbf{x}) b_{\alpha}^{\dagger}(\mathbf{p}) \right), \quad (13)$$

where we have defined

$$u_{\alpha}(\mathbf{p}) = u_{\sigma}(\mathbf{p}) |\tau\rangle, \quad (\alpha = \sigma, \tau), \quad \text{etc..}$$

The positive and negative spinors are given by

$$u_{\sigma}(\mathbf{p}) = \sqrt{\frac{E_p + M^*}{2E_p}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E_p + M^*} \end{pmatrix} \xi, \quad v_{\sigma}(\mathbf{p}) = \sqrt{\frac{E_p + M^*}{2E_p}} \begin{pmatrix} \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E_p + M^*} \\ 1 \end{pmatrix} \xi.$$

In the above equation, we have used abbreviations $E_p = \sqrt{M^{*2} + \mathbf{p}^2}$ and ξ for Pauli spinor. The Lorentz vector potential does not appear explicitly in the field. Using

these notations, we have for β_- and β_+ excitations in $N > Z$ nuclei, respectively,

$$\begin{aligned} F_-|0\rangle &= \sqrt{2} \int d^3p \sum_{\sigma\sigma'} \left(\bar{u}_\sigma(\mathbf{p}) \Gamma u_{\sigma'}(\mathbf{p}) a_{\sigma p}^\dagger(\mathbf{p}) a_{\sigma' n}(\mathbf{p}) \right. \\ &\quad \left. + \bar{u}_\sigma(\mathbf{p}) \Gamma v_{\sigma'}(-\mathbf{p}) a_{\sigma p}^\dagger(\mathbf{p}) b_{\sigma' n}^\dagger(-\mathbf{p}) \right) |0\rangle, \\ F_+|0\rangle &= \sqrt{2} \int d^3p \sum_{\sigma\sigma'} \bar{u}_\sigma(\mathbf{p}) \Gamma v_{\sigma'}(-\mathbf{p}) a_{\sigma n}^\dagger(\mathbf{p}) b_{\sigma' p}^\dagger(-\mathbf{p}) |0\rangle, \end{aligned}$$

Γ being $\Gamma = \gamma_5 \gamma_y$. The above equations show that there are excitations of proton-antineutron states in the β_- transitions in addition to particle-hole states, while neutron-antiproton states in the β_+ transitions. Using these equations, we obtain the total GT strengths for the β_- and β_+ transitions⁵,

$$\begin{aligned} \langle 0 | F_+ F_- | 0 \rangle &= 4 \frac{V}{(2\pi)^3} \int \frac{d^3p}{E_p^2} \left[\theta_{\mathbf{p}}^{(n)} \left(1 - \theta_{\mathbf{p}}^{(p)} \right) (M^{*2} + p_y^2) \right. \\ &\quad \left. + \left(1 - \theta_{\mathbf{p}}^{(p)} \right) (\mathbf{p}^2 - p_y^2) \right], \end{aligned} \quad (14)$$

$$\langle 0 | F_- F_+ | 0 \rangle = 4 \frac{V}{(2\pi)^3} \int \frac{d^3p}{E_p^2} \left(1 - \theta_{\mathbf{p}}^{(n)} \right) (\mathbf{p}^2 - p_y^2), \quad (15)$$

where we have defined the step function and the volume of the system as,

$$\theta_{\mathbf{p}}^{(i)} = \theta(k_i - |\mathbf{p}|) \text{ for } i = p \text{ and } n, \quad V = A (3\pi^2/2k_F^3),$$

k_n and k_p being Fermi momentum of neutrons and protons,

$$k_n^3 = \frac{2N}{A} k_F^3, \quad k_p^3 = \frac{2Z}{A} k_F^3. \quad (16)$$

The analytic expressions in Eqs.(14) and (15) for nuclear matter are useful for discussions of relativistic corrections to the β_- and β_+ transition strengths. As seen from the step functions in the square brackets of Eq.(14), the first term stems from the transitions of neutrons to proton states, while the second term is due to the transition of anti-neutrons to proton states. In the second term, the step function appears with a minus sign because of the Pauli principle. The part with the step function is called frequently Pauli blocking term. The integral of the first term is finite, but the one of the second term is infinite, although Pauli blocking term gives the finite value. For the β_+ transitions, Eq.(15) yields the matrix element coming from the transitions of anti-protons to neutron states only, since we assume nuclei with $N > Z$. The integral of the right hand side is also infinite.

The GT sum rule is obtained from the difference between Eqs.(14) and (15). Each of them is infinite, but the infinite terms exactly cancel each other and the difference becomes finite, yielding the GT sum rule value, as in Eq.(12)

$$\langle 0 | F_+ F_- | 0 \rangle - \langle 0 | F_- F_+ | 0 \rangle = 2(N - Z). \quad (17)$$

The sum rule is thus satisfied, if we take into account the $\bar{N}\bar{N}$ states.

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The $N\bar{N}$ states can not be excited by usual charge-exchange reactions, as mentioned before. If there is no coupling between particle-hole states and $N\bar{N}$ states, the only particle-hole states are observed at low excitation-energy region. Thus, the relativistic corrections come from the contribution of the $N\bar{N}$ states to Eq.(17). They are estimated by calculating the first term of Eq.(14) for the total β_- strength of the particle-hole states^{5,6},

$$S_{\text{ph}} = \frac{A}{k_F^3} \{Q(k_n) - Q(k_p)\}, \quad (18)$$

where $q(k_i)$ is defined as

$$Q(k_i) = \frac{k_i^3}{3} + 2k_i M^{*2} - 2M^{*3} \tan^{-1} \frac{k_i}{M^*}. \quad (19)$$

When we expand $Q(k_i)$ in terms of $(k_n - k_p)$, we obtain

$$S_{\text{ph}} \approx \left(1 - \frac{2}{3}v_F^2\right) 2(N - Z), \quad (20)$$

v_F being the Fermi velocity, $k_F/\sqrt{M^{*2} + k_F^2}$. The reduction factor in the first parentheses is the relativistic correction coming from the anti-nucleon degrees of freedom. When we employ the values as $v_F = 0.43$ which corresponds to the values $M^* = 0.6M$ and $k_F = 1.36 \text{ fm}^{-1}$ as in most relativistic models¹⁶, the sum rule value is quenched by about 12%. The quenched amount is taken by $N\bar{N}$ states,

$$[\langle 0 | F_+ F_- | 0 \rangle - \langle 0 | F_- F_+ | 0 \rangle]_{N\bar{N}} \approx \frac{2}{3}v_F^2 2(N - Z). \quad (21)$$

In finite nuclei, we can not obtain the analytic formulae, but can perform more sophisticated calculations numerically. We obtain the quenching of the sum rule value by 6.3% in ^{40}Ca with use of the NL-SH parameter set⁶, and by 7.7% in ^{90}Zr and by 8.4% in ^{208}Pb using the NL3⁹. These values are smaller than 12% in nuclear matter, since the effective mass in nuclear surface of finite nuclei is larger than that of nuclear matter. These results implies that the relativistic correction is not negligible in the discussion of the quenching phenomena of the GT sum rule value.

4. Relativistic RPA

When we discuss the excitation strengths, the particle-hole correlations should be examined as in non-relativistic models. Moreover, the coupling between the particle-hole states and $N\bar{N}$ states should be also investigated, since the present relativistic model has no reason why the anti-nucleon degrees of freedom can be neglected. The small coupling has a possibility to change the strengths of low lying states, because of the total strength of the $N\bar{N}$ states to be infinite. To estimate these effects, we study the GT response function of nuclear matter with the relativistic random phase approximation(RPA)^{17,18}. This framework requires two assumptions. First, we have to assume the coupling Lagrangian between particle-hole states and $N\bar{N}$ states. Second, we need a model to renormalize the divergence in the response function.

4.1. RPA without renormalization

For the coupling Lagrangian, we extend the non-relativistic models for the giant GT states. In non-relativistic models, the spin-isospin responses of nuclei are well described by the coupling Hamiltonian¹⁹,

$$V = \left[\left(\frac{f_\pi}{m_\pi} \right)^2 g' \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \left(\frac{f_\pi}{m_\pi} \right)^2 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{q^2 + m_\pi^2} - \left(\frac{f_\rho}{m_\rho} \right)^2 \frac{(\boldsymbol{\sigma}_1 \times \mathbf{q})(\boldsymbol{\sigma}_2 \times \mathbf{q})}{q^2 + m_\rho^2} \right] \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2. \quad (22)$$

The the first term with the Landau-Migdal parameter g' is responsible for the short-range part of the interaction, the second one is due to the pion exchange, and the last one comes from the rho-meson exchange. In nuclear matter, only the first term is responsible for the particle-hole interaction, since the momentum transfer is zero in the GT excitations. In relativistic models, we assume relativistic Landau-Migdal parameters which reduce to the non-relativistic one. Those are given by the pseudovector and tensor couplings¹¹,

$$\mathcal{L} = \frac{g_a}{2} \bar{\psi} \Gamma_i^\mu \psi \bar{\psi} \Gamma_{\mu i} \psi + \frac{g_t}{4} \bar{\psi} T_i^{\mu\nu} \psi \bar{\psi} T_{\mu\nu i} \psi, \quad (23)$$

where we have used the notations,

$$\Gamma_i^\mu = \gamma_5 \gamma^\mu \tau_i, \quad T_i^{\mu\nu} = \sigma^{\mu\nu} \tau_i. \quad (24)$$

In the non-relativistic limit, both terms in Eq.(23) reduce to the first term of Eq.(22) with the relationship,

$$g_a = g' \left(\frac{f_\pi}{m_\pi} \right)^2, \quad g_t = g' \left(\frac{f_\pi}{m_\pi} \right)^2. \quad (25)$$

The role of the tensor coupling Lagrangian has been discussed in ref.¹¹, and shown to have no contribution to the quenching problems. Therefore we neglect it from now on.

The relativistic RPA equation in nuclear matter can be solved in an analytic way. When we expand the excitation energy ω_{GT} and strength S_- of the GT state in terms of $(N - Z)$, we have⁸

$$\omega_{\text{GT}} \approx \frac{4k_F^3}{3\pi^2} \frac{g_a}{1 + 2\kappa'(\bar{N})g_a/(2\pi)^3} \left(1 - \frac{2}{3}v_F^2 \right) \frac{N - Z}{A}, \quad (26)$$

$$S_- \approx \frac{1}{(1 + 2\kappa'(\bar{N})g_a/(2\pi)^3)^2} \left(1 - \frac{2}{3}v_F^2 \right) 2(N - Z). \quad (27)$$

In the above equation, $\kappa'(\bar{N})$ stems from the coupling of the particle-hole states with $N\bar{N}$ states,

$$\kappa'(\bar{N}) = \frac{2}{3} \int d^3p \frac{\mathbf{p}^2}{E_p^3} \left(2 - \theta_p^{(n)} - \theta_p^{(p)} \right), \quad (28)$$

which is infinite, because of the first term of the parentheses. Thus, we need to renormalize the divergence in a proper way in order to obtain the finite value. Before discussing the renormalization, three comments will be in order.

First if we neglect the coupling between the particle-hole states and $N\bar{N}$ states, the excitation strength of the GT state becomes

$$S_- \longrightarrow \left(1 - \frac{2}{3}v_F^2\right) 2(N - Z), \quad (29)$$

which exhausts the GT sum value of the nucleon sector. Moreover, in the non-relativistic limit, Eqs.(26) and (29) reduce to

$$\begin{aligned} \omega_{\text{GT}} &\longrightarrow \frac{4k_F^3}{3\pi^2} g_a \frac{N - Z}{A}, \quad g_a = g' \left(\frac{f_\pi}{m_\pi}\right)^2, \\ S_- &\longrightarrow 2(N - Z), \end{aligned}$$

which were obtained previously in non-relativistic models²⁰.

Second, as seen in Eqs.(26), the coupling constant g_a is reduced by the repulsive coupling with $N\bar{N}$ as

$$g_a^{\text{eff}} = \frac{g_a}{1 + 2\kappa'(\bar{N})g_a/(2\pi)^3}. \quad (30)$$

The strength is also reduced by the coupling, as seen in Eq.(27). These forms are familiar in non-relativistic models where low lying states couple with high lying ones by a repulsive force²¹, although the value of $\kappa'(\bar{N})$ is positive infinite.

Third comment is rather serious for the approximation which neglects simply the divergence terms, as in the no-sea approximation¹². This approximation has been extensively used in previous calculations to avoid the renormalization^{5,8,9,13}. If the first term of Eq.(28) is neglected in order to avoid the divergence and the second and the third term are kept, the positive $\kappa'(\bar{N})$ is replaced by the negative $\kappa(\bar{N})$,

$$\kappa(\bar{N}) = -\frac{2}{3} \int d^3p \frac{p^2}{E_p^3} \left(\theta_p^{(n)} + \theta_p^{(p)} \right). \quad (31)$$

Therefore, in this approximation, the repulsive coupling is replaced artificially by the attractive one. As a result, the excitation energy becomes higher and the strength is enhanced due to the coupling in spite of the fact that the coupling Hamiltonian is repulsive.

These unphysical results are also seen in the relativistic Landau-Migdal parameter F_0 obtained previously from the same approximation in the σ - ω model^{22,23}. When the coupling of the particle-hole states with the $N\bar{N}$ states is neglected, the σ - ω model provides us with F_0 as

$$F_0 = F_v - (1 - v_F^2)F_s, \quad (32)$$

where we have defined

$$F_v = N_F \frac{g_v^2}{m_v^2}, \quad F_s = N_F \frac{g_s^2}{m_s^2}, \quad N_F = \frac{2k_F E_F}{\pi^2}, \quad E_F = \sqrt{k_F^2 + M^{*2}}. \quad (33)$$

In the above equations, $m_{v(s)}$ and $g_{v(s)}$ stand for the $\omega(\sigma)$ -meson mass and the coupling constant of the $\omega(\sigma)$ -meson with the nucleon, respectively. When we take into account the coupling with $N\bar{N}$ states by neglecting the divergence terms and keeping the Pauli blocking terms, the Landau-Migdal parameter F_0 becomes

$$F_0 = F_v - \frac{1 - v_F^2}{1 + a_s F_s} F_s, \quad (34)$$

where we have used the abbreviations,

$$a_s = \frac{3}{2} \left(1 - \frac{2}{3} v_F^2 + \frac{1 - v_F^2}{2v_F} \ln \frac{1 - v_F}{1 + v_F} \right) \approx \frac{1}{5} v_F^4 \left(1 + \frac{3}{7} v_F^2 + \dots \right). \quad (35)$$

Thus, the attractive part due to the σ -meson exchange is un-physically reduced due to the coupling with the $N\bar{N}$ states by the factor $(1 + a_s F_s) > 0$, in spite of the fact that the attractive part should be enhanced by the coupling. The Landau-Migdal parameter F_0 dominates the excitation energy of the giant monopole states. The reduction of the attractive force yields the increase of the excitation energy²³. Recent numerical calculations of the excitation energy have also shown this fact, although the experimental values are well reproduced¹³.

It is worthwhile noting one more comment on the approximation to neglect the divergence terms. Even if one neglects the divergence terms, the GT sum rule is satisfied, since the infinite terms in Eqs.(14) and (15) exactly cancel each other. The analytic formulae in Eqs. (14) and (15), however, show that in this approximation, the β_{\pm} transition strengths of $N\bar{N}$ states become negative. This fact changes the effect of the repulsive interaction into that of attractive one un-physically. We have also the negative strengths above 1GeV in the RPA calculations¹⁰. Thus the sum rule is not enough for justification of the approximation.

4.2. RPA with renormalization

Now let us discuss the renormalization of the divergence on the first term of Eq.(28). For this purpose, we employ n -dimensional regularization method²⁴. The 3-dimensional integral of the first term came from the 4-dimensional one,

$$\frac{2}{3} \int d^3 p \frac{p^2}{E_p^3} 2 = i \frac{2}{\pi} \int d^4 p \frac{M^* + p^2 + 2p_y^2}{(p^2 - M^{*2} + i\epsilon)^2}. \quad (36)$$

The above 4-dimensional integral is extended to the n -dimensional integral,

$$I_n = i \frac{2}{\pi} \int d^n p \frac{M^* + p^2 + 2p_y^2}{(p^2 - M^{*2} + i\epsilon)^2}. \quad (37)$$

In the limit $n \rightarrow 4$, we can separate the integral into the finite part and infinite part,

$$I_{n \rightarrow 4} = 4\pi \{ M^{*2} \Gamma(1 - n/2) |_{n \rightarrow 4} + 2M^{*2} \ln M^* \}, \quad (38)$$

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where the divergence is included in the Γ function. Since the Γ function is multiplied by M^{*2} ,

$$M^{*2} = (M - U_s)^2 = M^2 - 2MU_s + U_s^2,$$

we assume the 3 counter terms to cancel the divergence in the Lagrangian²⁵,

$$\Delta\mathcal{L}_c = \frac{1}{2} \left\{ a_0 + a_1\sigma + \frac{1}{2}a_2\sigma^2 \right\} \mathbf{A}_\mu \cdot \mathbf{A}^\mu. \quad (39)$$

In the above equation, σ and \mathbf{A}_μ denote the Lorentz scalar and pseudovector fields, respectively, and the coefficients a_i are determined so as to cancel the divergence. Finally, we obtain the renormalized $\kappa'(\bar{N})$ as

$$\begin{aligned} \kappa'(\bar{N})_{\text{ren}} &= -4\pi M^2 \left\{ 2\frac{M^{*2}}{M^2} \ln \frac{M}{M^*} + \left(1 - \frac{M^*}{M}\right) \left(1 - 3\frac{M^*}{M}\right) \right\} - \frac{16\pi}{15} k_F^2 v_F^3 \\ &\approx -4\pi M^2 \frac{2}{3} \left(1 - \frac{M^*}{M}\right)^3 - \frac{16\pi}{15} k_F^2 v_F^3. \end{aligned} \quad (40)$$

The first term of the right hand side is obtained by the renormalization of the divergence, and the last term comes from the Pauli blocking one. It should be noted that the above $\kappa'(\bar{N})_{\text{ren}}$ is negative for $M^* < M$, while $\kappa'(\bar{N})$ in Eq.(28) is positive infinite. Thus, the renormalization by the counter terms change the repulsive contribution from $N\bar{N}$ states into the attractive one to Eq.(30). This change happens in other cases, for example, in the renormalization of the divergence in the Landau-Migdal parameter F_0 ²³.

Let us estimate the quenching of the GT strength by using $g' = 0.6$, $M^* = 0.7306M$ and $k_F = 1.3 \text{ fm}^{-1}$. The first value is determined so as to reproduce the excitation energy of the GT state³, while the last two ones are obtained in the renormalized Hartree approximation of the $\sigma - \omega$ model to explain the binding energy of nuclear matter¹⁸. Finally we obtain

$$S_- = \frac{1 - 2v_F^2/3}{\{1 + 2\kappa'(\bar{N})_{\text{ren}}g_a/(2\pi)^3\}^2} 2(N - Z) \approx 2(N - Z). \quad (41)$$

Thus, the renormalization is very important, and almost cancels the quenching in the Hartree approximation. For this result, the first term in Eq.(40) is important, and second term from the Pauli blocking one is negligible. In fact, if we neglect simply the divergence terms in the RPA, we have

$$S_- = \frac{1 - 2v_F^2/3}{\{1 + 2\kappa(\bar{N})g_a/(2\pi)^3\}^2} 2(N - Z) \approx \left(1 - \frac{2}{3}v_F^2\right) 2(N - Z), \quad (42)$$

as shown in refs.⁵ and ⁶.

We note that $\kappa'(\bar{N})_{\text{ren}}$ has a rather strong density-dependence through the effective mass. In finite nuclei, the effect of $\kappa'(\bar{N})_{\text{ren}}$ on S_- may be weakened. In high density nuclear matter, on the contrary, the value of S_- is expected to be more enhanced.

Before closing this subsection, two comments should be added. The relativistic RPA satisfies the GT sum rule. Second, the energy-weighted sum of the β_- and β_+ transition strengths is equal to the ground-state expectation value of the double commutator as to the GT operator and the present Hartree Hamiltonian⁸,

$$\begin{aligned} & \sum_n (E_n - E_0) |\langle n | F_- | \tilde{0} \rangle|^2 + \sum_n (E_n - E_0) |\langle n | F_+ | \tilde{0} \rangle|^2 \\ &= \langle 0 | [F_+, [H, F_-]] | 0 \rangle, \end{aligned} \quad (43)$$

where E_n and E_0 stand for the excitation energies of the RPA excited state $|n\rangle$ and the ground state $|\tilde{0}\rangle$, respectively, and H denotes the Hartree Hamiltonian. The above relationship is well known in non-relativistic RPA for no-charge exchange excitations²⁶.

5. Conclusion

The Gamow-Teller (GT) sum rules with respect to the difference between β_- and β_+ transition strengths are investigated in the relativistic and non-relativistic theories. The sum rule value of the relativistic theory is the same as that of the non-relativistic theory, although each total strength of the β_- and β_+ transitions is infinite. In the non-relativistic theory, the sum rule value is exhausted by the particle-hole states, while in the relativistic theory, a part of the GT strengths are taken by the nucleon-antinucleon ($N\bar{N}$) states. Hence, if there is no coupling between the particle-hole states and $N\bar{N}$ states, the total GT strengths of the particle-hole states in the relativistic theory are reduced, compared to the sum rule value. According to the relativistic nuclear models developed for recent years^{15,16}, the quenching amount is estimated to be about 12% of the sum rule value in nuclear matter, and about 8% in finite nuclei.

There may be a possibility that the coupling of the particle-hole states can be neglected, from more fundamental reason beyond the present relativistic model. We have estimated, however, effects of the coupling on the GT strengths within the relativistic model, using the random phase approximation for nuclear matter. The divergence of the GT response function due to the $N\bar{N}$ states is properly renormalized by the n -dimensional regularization method²⁴. It has been shown that the coupling reduces the quenching of the GT strengths. In nuclear matter with the normal density, it is expected that the total strength of the β_- transition to the low lying states becomes nearly equal to the GT sum rule value. Since the coupling effect is density-dependent, it may be weakened in finite nuclei. In this sense, we expect that 8% is the upper limit of the relativistic correction in the present relativistic model.

We have also discussed the approximation like the no-sea approximation which neglects the divergent terms, but takes into account the Pauli blocking ones of the $N\bar{N}$ excitations. The approximation does not violate the GT sum rule, but yields negative excitation strengths for $N\bar{N}$ states. Because of this fact, the repulsive

interaction of the coupling works, as if it were an attractive one. We have the same problem in a description of the giant monopole states, where the attractive coupling interaction through the σ -meson exchange yields a repulsive effect. There seems to be no justification for the approximation to neglect the divergence.

Acknowledgments

The authors would like to thank Drs. N. Van Giai, Z.-Y. Ma and T. Maruyama for useful discussions.

References

1. K. Yako et al., Phys. Lett. B **615**, 193 (2005).
2. T. Suzuki, H. Sakai, and T. Tatsumi, in proceedings of the RCNP International Symposium on Nuclear Response and Medium Effects, edited by T. Noro et al., p.77 (Universal Academic Press, Tokyo, 1999).
3. T. Suzuki and H. Sakai, Phys. Lett. B **455**, 25 (1999).
4. A. Arima, et al., Phys. Lett. B **499**, 104 (2001).
5. H. Kurasawa, T. Suzuki and N. Van Giai, Phys. Rev. Lett. **91**, 062501 (2003).
6. H. Kurasawa, T. Suzuki and N. Van Giai, Phys. Rev. **C68**, 064311 (2003).
7. H. Kurasawa, T. Suzuki and N. Van Giai, Nucl. Phys. **A731**, 114 (2004).
8. H. Kurasawa and T. Suzuki, Phys. Rev. C **69**, 014306 (2004).
9. Z.-Y. Ma, et al., Euro. J. Phys. **A20**, 429 (2004).
10. H. Kurasawa and T. Suzuki, Phys. of Atomic Nucl. **67**, 1656 (2004).
11. T. Maruyama, H. Kurasawa and T. Suzuki, Prog. Theor. Phys. **113**, 355 (2005).
12. J. F. Dawson and R. J. Furnstahl, Phys. Rev. **C42**, 2009 (1990).
13. Z.-Y. Ma et al., Nucl. Phys. **A722**, C491 (2003), and references therein.
14. F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992).
15. B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).
16. P. Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996).
17. H. Kurasawa and T. Suzuki, Nucl. Phys. **A445**, 685 (1985).
18. H. Kurasawa and T. Suzuki, Nucl. Phys. **A490**, 571 (1988).
19. S. -O. Bäckman, G. E. Brown, and J. A. Niskanen, Phys. Rep. **24**, 1 (1985).
20. T. Suzuki, Nucl. Phys. **A379**, 110 (1982).
21. T. Suzuki and M. Kohno, Prog. Theor. Phys. **68**, 690 (1982).
22. H. Kurasawa and T. Suzuki, Nucl. Phys. **A454**, 527 (1986).
23. H. Kurasawa and T. Suzuki, Phys. Lett. **B474**, 262 (2000).
24. S. A. Chin, Ann. Phys. (N.Y.) **108**, 301 (1977).
25. T. Maruyama, H. Kurasawa and T. Suzuki, to be published.
26. D. J. Thouless, Nucl. Phys. **22**, 78 (1961).